Extreme Value of Functions

-One of the most useful things we can use the derivative for.

-"What is the most effective dose of medicine?"

Absolute Extreme Values

Let f be a function with domain D. Then $f(c)$ is the:

absolute maximum value on D iff $f(x) \le f(c)$ for all x in D.

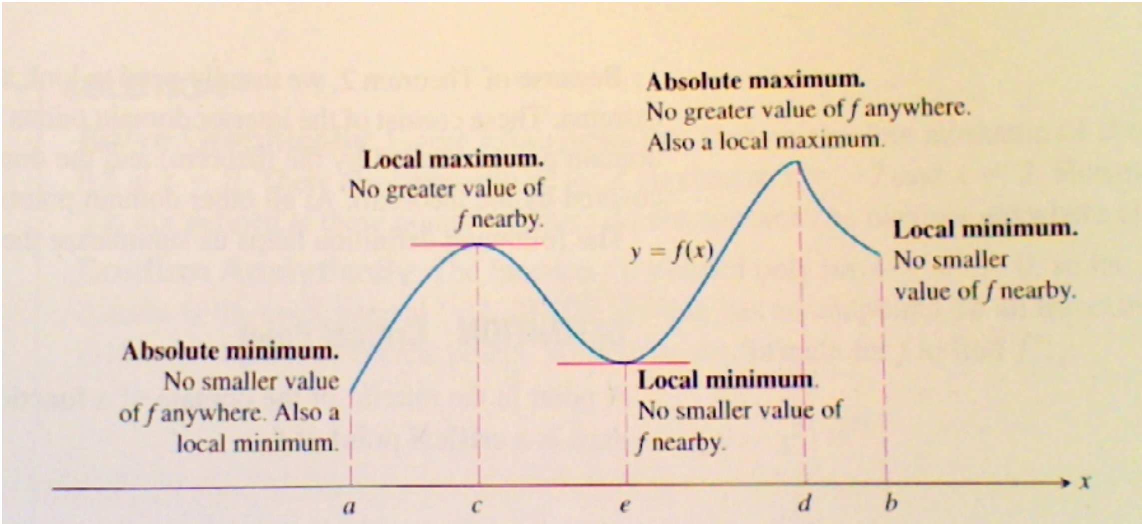
absolute minimum value on D iff $f(x) \ge f(c)$ for all x in D.

-Absolute or global max and min values are also called **absolute extrema.**

Example

-Functions with the same function rule can have different extrema depending on the domain.

Function Rule	Domain D	Absolute Extrema
$y = x^2$	$(-\infty, \infty)$	No abs. max abs. min of 0 at $x = 0$
$y = x^2$	$[0,2]$	abs max of 4 at $x = 2$ abs min of 0 at $x = 0$
$y = x^2$	$(0,2]$	abs. max of 4 at $x = 2$ No abs. min.
$y = x^2$	$(0,2)$	No abs extrema



-A list of local extrema will automatically include absolute extrema if there are any.

<u>Finding Extreme Values</u>

-Functions have local extreme values if f' is 0 or f' does not exist.

<u>Theorem-Local Extreme Values</u>

-If a function f has a local max value or a local min value at an interior point c of its domain, and f' exist at c then

$$f'(c) = 0$$

<u>Definition-Critical Point</u>

-A point in the interior of the domain of a function f at which $f' = 0$ or f' does not exist is a **<u>critical point</u>** of f.

<u>Example-Finding Absolute Extrema</u>

-Find the abs. max and min of $f(x) = x^{2/3}$ on $[-2,3]$.

$$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$$

-The derivative has no zeros but is undefined at $x = 0$.

-We must consider the one critical point and the endpoints:

Critical point value: $f(0) = 0$

Endpoint values: $f(-2) = (-2)^{2/3} = \sqrt[3]{4}$

$$f(3) = (3)^{2/3} = \sqrt[3]{9}$$

-We can see from this that the functions abs. max value is $\sqrt[3]{9} \approx 2.08$ and occurs at the right endpoint $x = 3$.

-The abs. min value is 0 and occurs at the interior point $x = 0$.

-Graph to confirm.

Example-Finding Extreme Values

$$f(x) = \frac{1}{\sqrt{4 - x^2}}$$

-The function is only defined on $4 - x^2 > 0$ so the domain is $(-2,2)$.

-The domain as no endpoints.

-Find the derivative

$$f(x) = \frac{1}{\sqrt{4-x^2}} = \left(4-x^2\right)^{-1/2}$$

$$f'(x) = -\frac{1}{2}\left(4-x^2\right)^{-3/2}(-2x) = \frac{x}{\left(4-x^2\right)^{3/2}}$$

-The only critical point in the domain $(-2,2)$ is $x = 0$.

$$f(0) = \frac{1}{\sqrt{4-(0)^2}} = \frac{1}{2}$$

-This is the only candidate for an extreme value.

-To tell whether $1/2$ is an extreme value examine the function

$$f(x) = \frac{1}{\sqrt{4-x^2}}$$

-As x moves away from zero on either side the denominator gets smaller, the value of f increases.

-We have a min value at $x = 0$, the min is absolute.

-Graph to confirm.